On Measuring the Efficiency of Monetary Policy

Walter Briec,* Emmanuelle Gabillon,† Laurence Lasselle‡
and Hermann Ratsimbanierana§

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Abstract

Cecchetti et al. (2006) develop a method for allocating macroeconomic performance changes among the structure of the economy, variability of supply shocks and monetary policy. We propose a dual approach of their method by borrowing well-known tools from production theory, namely the Farrell measure and the Malmquist index. Following Färe et al (1994) we propose a decomposition of the efficiency of monetary policy. It is shown that the global efficiency changes can be rewritten as the product of the changes in macroeconomic performance, minimum quadratic loss, and efficiency frontier.

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*Université de Perpignan, Département d’Économie, 52 Avenue Villeneuve, 66860 Perpignan Cedex, France.
†Université Montesquieu - Bordeaux IV, France, GREThA UMR - CNRS 5113
‡(corresponding author) University of St. Andrews, School of Economics and Finance, St. Andrews, Fife, KY16 9AL, U.K.; E-mail: LL5@st-andrews.ac.uk
§Université de Perpignan, Département d’Économie, France
1 Introduction

Cecchitti et al. (2006, p. 409) [CFLK thereafter] ”develop a method for measuring the contribution of improved monetary policy to observed changes in macroeconomic performance and then use it to explain the observed increase in macroeconomic stability in a cross-section of countries. [Their] technique involves examining changes in the variability of inflation and output over time. (...) [They] compute the output-inflation variability frontier describing the best outcomes that a policy-maker can hope to achieve. Movements toward this frontier are interpreted as improvements in monetary policy efficiency.”

In this paper, we propose an alternative method for measuring these movements by borrowing two production theory traditional tools, namely the Farrell measure and the Malmquist index. The Farrell measure\(^1\) evaluates firms’ performance. In the case of constant returns to scale technology, this distance function allows to find a more efficient way of producing the same level of output given that in the new allocation the inputs are employed in the same ratio as in the original (but inefficient) allocation.\(^2\) This ratio is often called the measure of technical efficiency (see Farrell (1957)). However, it does not ensure that the firm is operating with ”economic” efficiency as the cost of producing the output may not be minimized. This is the reason Farrell constructs another ratio, often referred as the measure of allocative efficiency. This is the ratio of the cost of the allocation minimizing the production cost to that of the technically efficient allocation. The firm’s efficiency is then measured by the product of both efficiency measures.

Our method first rests on the use of the Farrell measure in the CFLK’s framework. We show that this measure is dual to the loss function of the policy-maker. We retrieve the optimal policy efficiency parameter by considering the minimised loss function. As a result, we can distinguish the policy efficiency and the allocative efficiency. We then derive the Malmquist index\(^3\) in our framework. This second tool was named after Malmquist (1953) who defined it in a consumer theory context. It was later specified by Caves et al. (1982) in the production theory context. Adapted in our framework, it is defined from Farrell measures evaluating the output-inflation variability frontier and determines the shift of this frontier and the changes in macroeconomic performance. Along this line we propose a decomposition inspired from the work by Färe et al. (1994). In their paper they show how to decompose the Malmquist productivity index into two component measures, namely, technical change and efficiency change. In this paper, it is shown that the global efficiency changes can be rewritten as the product of the changes in macroeconomic performance, minimum quadratic loss, and efficiency frontier. As in

\(^1\)Introduced by Farrell (1957), it is the inverse of the Shephard distance function (see Shephard (1970)).

\(^2\)See Cornes (1992, pp. 133-4) for a detailed presentation.

\(^3\)Note that the Farrell measure is a Malmquist index.
CFLK, we can identify the contributions of improvements in the efficiency of monetary policy and changes in the variability of aggregate supply shocks.

Our paper unfolds as follows. Section 2 introduces the basic building blocks of our method in CFLK’s framework. It introduces the Farrell measure and formulates a general principle of duality between the quadratic loss function and the macroeconomic performance measure. Section 3 completes the presentation of our technique by defining the Malmquist index and linking it to these blocks and the various efficiency measures. Section 4 concludes.

2 Measuring Efficiency of Monetary Policy: the Farrell Measure Approach

Consider a simple economy in which the monetary authority faces a trade-off between the variability of output and that of inflation. “This trade-off allows to construct an efficiency frontier that traces the point of minimum inflation and output variability (...) [see Figure 1]. The location of the efficiency frontier \( \mathcal{EF} \) depends on the variability of aggregate supply shocks - the smaller such variability, the closer the frontier is determined by the structure of the economy. If monetary policy is optimal, the economy will be on this curve. The exact point depends on the policy-maker’s preferences for inflation and output stability. When policy is sub-optimal, the economy will not be on this frontier. Instead, the performance point \( V \) will be up and to the right with inflation and output variability both in excess of other feasible points. Movements of \( V \) toward the frontier are an indication of improved policy-making.” (CFLK, p. 412).

![Figure 1 Efficiency, Allocative Efficiency and Duality](image)
We define by \( V_t \) the inflation-output variability set at period \( t \):

\[
V_t = \{(\text{Var}(\pi_t), \text{Var}(y_t)) : \pi_t \in \Pi_t, y_t \in Y_t\}
\]  

(2.1)

where \( \pi_t \) and \( y_t \) are respectively inflation and output at period \( t \), \( \Pi_t \) and \( Y_t \) the set of all the inflation rates at period \( t \) and that of output at period \( t \), and \( \text{Var} \) the variance which measures the variability of either inflation, or output. Let us assume that at each period \( t \): \( V_t \) is closed; \( V_t \) is convex; and \( V_t \) satisfies the free variability assumption, i.e. if \((V_{\pi_t}, V_{y_t}) \in V_t\), then \((\bar{V}_{\pi_t}, \bar{V}_{y_t}) \geq (V_{\pi_t}, V_{y_t}) \Rightarrow (\bar{V}_{\pi_t}, \bar{V}_{y_t}) \in V_t\), where \( V_{\pi_t} \) and \( V_{y_t} \) are the coordinates of point \( V \) and \( \bar{V}_{\pi_t} \) and \( \bar{V}_{y_t} \) those of \( \bar{V} \). These three assumptions will enable us to characterize the efficiency frontier of \( V_t \) by using the Farrell measure. In our framework, the Farrell measure is defined by:

\[
\mathcal{F}_t(\pi_t, y_t) = \min_{\delta} \{\delta : (\delta \text{Var}(\pi_t), \delta \text{Var}(y_t)) \in V_t\}
\]  

(2.2)

As we shall see, \( \delta \) can reflect the efficiency of policy-making. The principle of the Farrell measure is illustrated in Figure 1. \( V^* \) represents an efficient allocation of policy-making and is found from the projection on the efficiency frontier by homothety of the (inefficient) performance point \( V \). We then have:

\[
\mathcal{F}_t(\pi_t, y_t) = \frac{|OV^*|}{|OV|}
\]  

(2.3)

The performance point \( V \) is then:

\[
(\text{Var}(\pi_t)^*, \text{Var}(y_t)^*) = \mathcal{F}_t(\pi_t, y_t)(\text{Var}(\pi_t), \text{Var}(y_t))
\]  

(2.4)

We define by \( \mathcal{E}F_t \) the efficiency frontier,\(^4\) the set of all efficient points in the variability set at period \( t \). The Farrell measure of each of these efficient points is equal to one. Therefore:

\[
\mathcal{E}F_t = \{(\text{Var}(\pi_t), \text{Var}(y_t)) \in V_t : \mathcal{F}_t(\pi_t, y_t) = 1\}
\]  

(2.5)

As CFLK (p. 412-3), we assume that the objective of the policy-maker is to minimise a weighted sum of inflation and output variability, summarised by the standard quadratic loss function as:

\[
\text{Loss} = \lambda \text{Var}(\pi) + (1 - \lambda) \text{Var}(y)
\]  

(2.6)

where \( \lambda \in [0,1] \) is the policy-maker’s preference parameter. However, in contrast to CFLK, we are not going to assume that \( \lambda \) is constant at this stage.

At each period \( t \), we define the macroeconomic performance by \( P_t(\lambda_t, \pi_t, y_t) \). It is the weighted average of the observed variability of inflation and output:

\[
P_t(\lambda_t, \pi_t, y_t) = \lambda_t \text{Var}(\pi_t) + (1 - \lambda_t) \text{Var}(y_t)
\]  

(2.7)

\( ^4 \)Also called output-inflation variability frontier.
Note that $P_t(\lambda_t, \pi_t, y_t) > 0$. We define the minimum quadratic loss function by $ML_t(\lambda_t)$ at each $t$:

$$ML_t(\lambda) = \inf_{\lambda_t} \{ P_t(\lambda_t, \pi_t, y_t) : (\text{Var}(\pi_t), \text{Var}(y_t)) \in \mathcal{Y}_t \}$$  \hspace{1cm} (2.8)

Therefore, the minimum value function for the policy-maker’s decision is simply determined by her preference parameter. Knowledge of this parameter allows the selection of a unique point belonging to the efficient frontier which minimizes the policy-maker’s quadratic loss function.

Before going further, let us remind that we propose an alternative method for measuring the policy-maker’s efficiency. We depart from CFKL on two aspects.

First, as we hinted earlier, the Farrell measure is going to reflect the policy-making efficiency. However, CFKL consider policy inefficiency. The latter, denoted by $E_t$, is the difference between the macroeconomic performance and the variability of supply shocks denoted by $S_t$:

$$E_t = P_t(\lambda_t, \pi_t, y_t) - S_t(\lambda_t, \pi_t, y_t)$$  \hspace{1cm} (2.9)

In our case, $S_t$ is:

$$S_t(\lambda_t, \pi_t, y_t) = \lambda_t \text{Var}(\pi_t)^* + (1-\lambda_t) \text{Var}(y_t)^* = \delta^* \left[ \lambda \text{Var}(\pi_t) + (1-\lambda) \text{Var}(y_t) \right]$$  \hspace{1cm} (2.10)

where $\delta^* = \mathcal{F}_t(\pi_t, y_t)$. In other words, we have:

$$S_t(\lambda_t, \pi_t, y_t) = \mathcal{F}_t(\pi_t, y_t) P_t(\lambda_t, \pi_t, y_t)$$  \hspace{1cm} (2.11)

and $E_t$ becomes:

$$E_t = (1 - \mathcal{F}_t(\pi_t, y_t)) P_t(\lambda_t, \pi_t, y_t)$$  \hspace{1cm} (2.12)

Second, as our method borrows tools from production theory, we aim at deriving a multiplicative form of policy efficiency and not an additive form as proposed by CFLK. Hence we define policy efficiency by $PE_t$

$$PE_t(\pi_t, y_t) = \frac{S_t(\lambda_t, \pi_t, y_t)}{P_t(\lambda_t, \pi_t, y_t)} = \mathcal{F}_t(\pi_t, y_t)$$  \hspace{1cm} (2.13)

Our measure does not depend on the policy-maker’s preference parameter, $\lambda$, which is an advantage. By combining (2.12) and (2.13), we obtain a link between the two measures:

$$E_t = (1 - PE_t(\pi_t, y_t)) P_t(\lambda_t, \pi_t, y_t)$$  \hspace{1cm} (2.14)

When the pair $(\pi_t, y_t)$ is efficient, $PE_t(\pi_t, y_t) = 1$ and $E_t = 0$, i.e. there is no inefficiency. When $PE_t$ is increasing (resp. decreasing), then $E_t$ is decreasing (increasing).
Characterizing the policy-maker’s preference parameter at each period \( t \) rests on an analytical programme in which the minimum quadratic loss function plays a crucial role. Our technique is inspired from the duality concept initiated by Shephard (1953) in production theory. To grasp the dual relationship between the quadratic loss function and the macroeconomic performance at period \( t \), we first define the Overall Efficiency by:

\[
OE_t(\lambda_t, \pi_t, y_t) = \frac{ML_t(\lambda_t)}{P_t(\lambda_t, \pi_t, y_t)}
\]

(2.15)

Thus, the overall efficiency is the ratio of the minimum quadratic loss function to the observed macroeconomic performance. Note that one has \( OE_t(\lambda_t, \pi_t, y_t) \leq 1 \).

As in production theory, this overall efficiency can be geometrically decomposed into two components: the Policy Efficiency computed from the Farrell measure \( F_t(\pi_t, y_t) \) and the Allocative Efficiency denoted as \( AE_t(\lambda_t, \pi_t, y_t) \). Therefore:

\[
OE_t(\lambda_t, \pi_t, y_t) = F_t(\pi_t, y_t).AE_t(\lambda_t, \pi_t, y_t)
\]

(2.16)

Or equivalently:

\[
OE_t(\lambda_t, \pi_t, y_t) = P E_t(\pi_t, y_t).AE_t(\lambda_t, \pi_t, y_t)
\]

(2.17)

Recall that the Farrell measure on the efficiency frontier equals one. However, reaching a point on the efficiency frontier does not necessarily imply that the point minimizing the quadratic loss function has been reached. It is in this sense that our notion of policy efficiency is similar to the notion of technical efficiency in production theory. Allocative efficiency measures the needed reallocation, along the efficiency frontier, to achieve the minimum of the quadratic loss function. This requires a re-adjustment of the policy-maker’s preference parameter. When the overall efficiency equals one, the point belonging to the efficiency frontier minimizing the quadratic loss has been determined.

Since for \( \lambda_t \in [0, 1] \), \( V_t \) is a subset of \( \{(\text{Var}(\pi_t), \text{Var}(y_t)) : P_t(\lambda_t, \delta \pi_t, \delta y_t) \geq ML_t(\lambda_t)\} \), we deduce \( OE_t(\lambda_t, \pi_t, y_t) \leq F_t(\pi_t, y_t) \). Thus we have \( AE_t(\lambda_t, \pi_t, y_t) \leq 1 \). However, by applying the weak version of the separation theorem, we find some \( \lambda_t^* \in [0, 1] \) such that \( P_t(\lambda_t^*, \delta \pi_t, \delta y_t) = ML_t(\lambda_t^*) \). Therefore:

\[
\bar{F}_t(\pi_t, y_t) = \max_{\lambda_t} \left\{ \frac{ML_t(\lambda_t)}{P_t(\lambda_t, \delta \pi_t, \delta y_t)} \right\}
\]

(2.18)

This formula yields a program to compute the preference parameter. Therefore, we define the policy-maker’s adjusted preference function:

\[
\lambda_t^*(\pi_t, y_t) = \arg \max_{\lambda_t^*} \left\{ \frac{ML_t(\lambda_t^*)}{P_t(\lambda_t^*, \delta \pi_t, \delta y_t)} \right\}
\]

(2.19)

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6Called ”technical efficiency” in production theory.
Our procedure has allowed us to derive analytically the policy-maker’s preference parameter.

In Figure 1, the overall efficiency and allocative efficiency are respectively $OE_t = \frac{\|OV_t\|}{\|OV\|}$ and $AE_t = \frac{\|OV_t\|}{\|OV\|}$. The ratio $\frac{\lambda_t}{1-\lambda_t}$ represents the slope of the efficient frontier at the optimal point $V^*$. If we denote as $PE_t = \overline{z}_t(\pi_t, y_t)$ the efficiency of policy-making, we retrieve the decomposition described in (2.17). The efficiency of policy-making can therefore be characterized from the Farrell measure. At each period $t$, it can be computed from the ratio:

$$PE_t(\pi_t, y_t) = \frac{ML_t(\lambda_t^*)}{P_t(\lambda_t^*, \delta\pi_t, \delta y_t)} = \lambda_t^*\text{Var}(\pi_t)^* + (1 - \lambda_t^*)\text{Var}(\delta\pi_t)^* + \lambda_t^*\text{Var}(y_t)^* + (1 - \lambda_t^*)\text{Var}(\delta y_t)^*$$ (2.20)

However, because of our forthcoming use of the Malmquist index (which is based upon a geometric mean of distance functions), we have provided a formula of policy efficiency using ratios rather than differences as in CFLK.

Before going further, let us compare the two approaches. CFLK introduce a measure of the change in performance to calculate the proportion that can be accounted for by improved policy. In our notations, their measure is given by:

$$ML_t = \frac{\Delta E}{\Delta P} = \frac{E_t - E_{t-1}}{P_t - P_{t-1}}$$ (2.21)

Our approach is slightly different although it is strongly connected. In particular, the Malmquist index we shall refer to involves intertemporal comparison, i.e. observations related in $t$ are compared to the efficiency frontier in $t+1$ and conversely. For this purpose, we define the change in macroeconomic performance as the change from one period to the next period:

$$\Delta P_t = \frac{P_{t+1}(\lambda_{t+1}, \pi_{t+1}, y_{t+1})}{P_t(\lambda_t^*, \pi_t, y_t)}$$ (2.22)

This change takes into account the policy-maker preference parameter $\lambda^*$ corresponding to the optimal policy for the period.

3 Malmquist Index, Policy Efficiency Change and Efficient Frontier Shift

In production theory, the Malmquist Index is a bilateral index which compares the production technology of two economies. In our framework, it measures the contribution of improved policy to observed changes in macroeconomic performance between two periods.

Recall that we previously denoted the Farrell measure as:

$$\overline{z}_b(\pi_a, y_a) = \min\{\delta : (\delta\text{Var}(\pi_a), \delta\text{Var}(y_a)) \in V_b\}$$ (3.1)
We define by $M(\pi_t, y_t, \pi_{t+1}, y_{t+1})$ the Malmquist index:

$$M(\pi_t, y_t, \pi_{t+1}, y_{t+1}) = \left[ \frac{\mathcal{F}_t(\pi_t, y_t)}{\mathcal{F}_t(\pi_{t+1}, y_{t+1})} \cdot \frac{\mathcal{F}_{t+1}(\pi_t, y_t)}{\mathcal{F}_{t+1}(\pi_{t+1}, y_{t+1})} \right]^{\frac{1}{2}} \quad (3.2)$$

It is the geometric mean\(^7\) of two *Global Efficiency Changes* at $t+1$ and $t$. Each change is said to be "global" as it simultaneously combines inflation and output variability. If $(\pi_{t+1}, y_{t+1})$ improves the performance regarding to the efficiency frontier in $t$ and the relative performance of $(\pi_t, y_t)$, then $\mathcal{F}_t(\pi_{t+1}, y_{t+1}) > \mathcal{F}_t(\pi_t, y_t)$. It implies that the first ratio of the Malmquist index is less then 1. Symmetrically, if the shift of the efficiency frontier involves a better performance regarding to $(\pi_t, y_t)$ then $\mathcal{F}_{t+1}(\pi_{t+1}, y_{t+1}) > \mathcal{F}_{t+1}(\pi_t, y_t)$. The second ratio is also less then 1. The Malmquist index is therefore less than 1. By comparing the observations and the efficiency frontiers in $t$ and $t+1$, we deduce that the performance has improved.

As in production theory, our Malmquist index can be decomposed into two components. The first component represents the *Policy Efficiency Change* between two successive periods and is defined by:

$$\Delta PE_t = \frac{\mathcal{F}_t(\pi_t, y_t)}{\mathcal{F}_{t+1}(\pi_{t+1}, y_{t+1})} \quad (3.3)$$

The second component, related to the shift of the efficiency frontier (Frontier Change), is defined by:

$$\Delta FC_t = \left[ \frac{\mathcal{F}_{t+1}(\pi_t, y_t)}{\mathcal{F}_t(\pi_t, y_t)} \cdot \frac{\mathcal{F}_{t+1}(\pi_{t+1}, y_{t+1})}{\mathcal{F}_{t+1}(\pi_{t+1}, y_{t+1})} \right]^{\frac{1}{2}} \quad (3.4)$$

Indeed, the ratios $\frac{\mathcal{F}_{t+1}(\pi_t, y_t)}{\mathcal{F}_t(\pi_t, y_t)}$ and $\frac{\mathcal{F}_{t+1}(\pi_{t+1}, y_{t+1})}{\mathcal{F}_{t+1}(\pi_{t+1}, y_{t+1})}$ are respectively the shift of the efficiency frontier given points $(\pi_t, y_t)$ and $(\pi_{t+1}, y_{t+1})$. By combining (3.2), (3.3) and (3.4), we obtain:

$$M(\pi_t, y_t, \pi_{t+1}, y_{t+1}) = \Delta PE_t \cdot \Delta FC_t \quad (3.5)$$

\(^7\)The use of the geometric mean allows us to avoid an arbitrary selection among base years.
In Figure 2, (3.2) is given by

\[ M(\cdot) = \left[ \left| \frac{OV_t'}{OV_t} \right| \left| \frac{OV_{t+1}'}{OV_{t+1}} \right| \left| \frac{OV_t^*}{OV_{t+1}^*} \right| \left| \frac{OV_{t+1}'}{OV_{t+1}} \right| \right]^{\frac{1}{2}} \]

We then re-arrange (2.18) and (2.19) to obtain the decomposition of the policy efficiency change:

\[ \Delta PE_t = \Delta P_t \Delta ML_t \] (3.6)

where \( \Delta P_t \) is the performance change defined in (2.22) and

\[ \Delta ML_t = \frac{ML_t(\lambda_t^*(\pi_t, y_t))}{ML_{t+1}(\lambda_{t+1}^*(\pi_{t+1}, y_{t+1}))} \] (3.7)

Inserting (3.6) into (3.5):

\[ M(\pi_t, y_t, \pi_{t+1}, y_{t+1}) = \Delta P_t \Delta ML_t \Delta FC_t \] (3.8)

In other words, the global efficiency changes can be rewritten as the product of the changes in macroeconomic performance (\( \Delta P_t \)), minimum quadratic loss (\( \Delta ML_t \)), and efficiency frontier (\( \Delta FC_t \)). As in CFLK, we can identify the contributions of improvements in the efficiency of monetary policy (through movements of the performance point toward the efficiency frontier) and changes in the variability of aggregate supply shocks (through the shift of the efficiency frontier). In constrast to CFLK, we can derive analytically the policy-maker’s preference parameter and the different components of global efficiency changes.
4 Conclusion

Our short paper presented a method for measuring the contributions of improved monetary policy to observed changes in monetary policy. Our method borrows two production theory traditional tools, namely the Farrell measure and the Malmquist index. It differs from the method developed by CFLK in two respects. Firstly, we are able to derive analytically the policy-maker’s preference parameter at each period. Secondly, our technique specifies all components of efficiency measures. One could go further by applying it in a nonparametric context. It would then follow Farrell’s approach in which linear programming techniques are used.

References


